Sethood and situations

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In *Situations and Attitudes* [**1**983] Barwise and Perry decide that meaning should be taken to be a triadic relation not a dyadic one. While there may be good reasons for deciding to procede on this basis one of the reasons they give is definitely a bad one, and this is the principal subject of this note.

On pp 222-23 they remark that if meaning is to be a dyadic relation it is necessary that the complement of a situation should—at least sometimes—be another situation. In the set theory which is the basis for their development—KPU—it is elementary that the complement of a set is never a set. This compels the designers of situation semantics to make meaning a triadic relation as we will now explain.

In [**1**983], Barwise and Perry take *individuals*, *properties* and *relations*, and *locations* as primitives. A *situation-type* is a partial function from n-ary relations and n individuals to the set $\{0, 1\}$ (page 8). In modern situation semantic parlance, this is often referred to as an *infon*, or (more precisely) a *basic infon*. An *event*, or *course-of-events* (*coe*), is a function from locations to situation-types. For example, the situation-type corresponding to a (real-world) situation in which a dog named Molly barks would be

$$\langle \text{at } l, \text{barks}, \text{Molly}, 1\rangle$$

and one related *coe* might be

$$e = \{\langle \text{at } l, \text{barks}, \text{Molly}, 1\rangle, \langle \text{at } l', \text{shouts at}, \text{Mr.Levine}, \text{Molly}, 1\rangle,$$
$$\langle \text{at } l'', \text{barks}, \text{Molly}, 0\rangle\}$$

Consider the predicate SO ("seeing option") on *coe*s. In a given event, s, an individual, a, classifies events according to what s/he sees and knows. That is

$\langle SO, a, e_1, 1\rangle \in s$ if e_1 is compatible with what a sees and knows;

$\langle SO, a, e_2, 0\rangle \in s$ if e_2 is incompatible with what a sees and knows;

This is a partial classification of events; i.e. some events may be neither SO-yes nor SO-no.

Further to this,

- *Definition:* In a given situation, s, an event, e, is a *visual option* for agent a if $\langle SO, a, e, 1\rangle \in s$
- *Definition:* Similarly, e is a *visual alternative* for a if it is not the case that $\langle SO, a, e, 0\rangle \in s$

Given s as above, let

$$X_{VO} = \{ \text{ e: } \langle SO, a, e, 1\rangle \in s\}$$
= collection of events which are visual/seeing options for a in s.

Also, let

$$X_{NVO} = \{ \text{ e: } \langle SO, a, e, 0\rangle \in s\}$$
= collection of events which a classifies as *not* being visual options.

Then

$$X_{VA} = \text{collection of visual alternatives for } a = \overline{X_{NVO}}.$$

(In general, we cannot assume $\overline{X_{NVO}} = X_{VO}$.)

An utterance, ϕ, determines a triple

$$\Phi = \langle d, c, \phi \rangle$$

composed of a *discourse situation*, d, a *speaker connection function*, c, and the utterance, ϕ.

Furthermore, our *interpretation* relation (a function from utterances of the above form to collections of events) is given as

$$[\![\Phi]\!] = \text{interpretation of } \phi \text{ according to } d \text{ and } c = \{e : d, c[\![\phi]\!]e \text{ } holds\} \ .$$

The speaker connection function, c, (or *anchor*) grounds the individuals, relations, and locations mentioned in ϕ to actual entities participating in the discourse situation, d. $[\![*]\!]$ is thus a binary relation, relating the utterance triple to the "described situation", $[\![\Phi]\!]$.

For example, if ϕ = FIDO RAN, $c(FIDO)$ ["FIDO" is mentioned] = Fido ["Fido" is used], and $c(ran) = l$ [a location], then if

$$\langle l, ran, Fido, 1 \rangle \in e$$

we have $e \in [\![\Phi]\!]$.

There is a problem with this analysis that leads Barwise and Perry to seek a representation of mental states and events with which to augment the interpretation relation. The problem involves a distinction Barwise and Perry make between *epistemic* and *non-epistemic* perception. Attitudinal reports involving the phrase "see that" followed by a finite complement involve epistemic perception—that is, they yield information about the inference an agent has performed after seeing a given *coe* or situation (page 207). The problem comes about when Barwise and Perry attempt to characterize attitude reports involving "see that" in terms of the relation SO.

On pp. 209-11 Barwise and Perry claim

$$\text{a sees that } \phi \Longrightarrow \{e : not\ d, c[\![\phi]\!]e\} \subseteq X_{NVO}(\star)$$

i.e. those events not in the interpretation of ϕ must be classified as *SO*-no.

Proof: (p 211) "A situation e is one where a sees that ϕ if ϕ holds in each of a's visual alternatives at the appropriate location, l." That is,

$$a \text{ sees that } \phi \Longrightarrow X_{VA} \subseteq [\![\Phi]\!]$$

and since $X_{VA} \subseteq [\![\Phi]\!] \Leftrightarrow \overline{[\![\Phi]\!]} \subseteq X_{NVO}$ (taking complements), we obtain the result.

This yields the following *situational analysis of attitudinal reports involving epistemic perception.* Given an utterance

$$\sigma := \text{a SAW THAT } \phi,$$

in order for σ to describe an event, e, we use $(\star)$ to obtain that we must have at $l = c(SAW)$, for every event, e_1, either

$$e_1 \in [\![\Phi]\!]$$

or

$$\langle l, SO, A, e_1, 0\rangle \in e.$$

The fact that any *SO*-no event must be classified as such by the event e (the event corresponding to the attitude report) means that if we view e as a collection of infons, we will have

$$|e| \geq |X_{NVO}|$$

and by a result above, $|X_{NVO}| \geq \left|\overline{[\![\Phi]\!]}\right|$.

It is Barwise and Perry's contention (p. 222) that $\overline{[\![\Phi]\!]}$ is a proper class, therefore e is as well.

> ... there is a proper class of events e_1 in which Jackie was not biting Molly, events that must be classified with *SO*-no. But then [the event] e required to classify Joe's visual state must be a proper class. (p.222)

Thus such events cannot, for example, be constituents of other situations. This means that iterated attitude reports cannot be handled in this framework.

This can be rectified by adopting as a set-theoretic basis a set theory in which the complement of a set is always a set. Whatever reasons caused them to desire a set theory with urelements should presumably still be respected, so if we can find a consistent set theory with urelements and a universal set the outlook will be a lot brighter. Fortunately there is such a system, the Jensen-Quine system of set theory known as *NFU*. For more on this see Holmes [1994] or [1996]. Of course an easy consequence of an axiom of complementation such as we have in NFU is the negation of the axiom of foundation. Barwise has elsewhere [**1**984] argued that we should not regard the axiom of foundation as essential.

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1996 M. Randall Holmes, Naive Set Theory with a Universal Set, unpublished, available on the WWW at `<http://math.idbsu.edu/faculty/holmes.html>`