An Exercise in Structural Induction

Thomas Forster

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A set x is **transitive** if $z \in y \in x$ implies $z \in x$.
$\mathcal{P}(x)$ is $\{y : y \subseteq x\}$, the **power set** of x.
Define V_ω to be the $\subseteq$-smallest set A containing $\emptyset$ and containing $x \cup \{y\}$ whenever it contains x and y.

(i) Show that V_ω is closed under $\cup$.

(ii) Show that V_ω is closed under $\bigcup$.

(iii) Show that V_ω is the $\subseteq$-smallest set containing $\emptyset$ and closed under pairing (existence of $\{x, y\}$) and $\cup$.

(iv) Show that V_ω is the $\subseteq$-smallest transitive set containing $\emptyset$ and closed under $\mathcal{P}$.

(v) Show that V_ω is the $\subseteq$-smallest set containing all its finite subsets.