2010p1q4

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Let 'x', 'y', 'z' range over individuals I and 'a', 'b' range over societies S. Let 'M', 'F' and 'T' be atomic predicates as follows:

$M(x, a)$:	x is a member of society a;
$F(a)$:	society a involves fighting;
$T(x, y, a)$:	x talks to y about a;

(a) Formalise each of the following English statements and translate each of the following formulæ into idiomatic English (natural English sentences).

(i) $(\forall x, y, a)(T(x, y, a) \to T(y, x, a))$
(ii) Nobody talks to themselves about anything.
(iii) There's at most one society involving fighting.
(iv) All societies have at least two members.
(v) $(\forall a)((\exists x, y)(M(x, a) \wedge M(y, a) \wedge x \neq y) \to$
$(\exists x, y, b)(M(x, a) \wedge M(y, a) \wedge x \neq y \wedge T(x, y, b) \wedge F(b)))$
(vi) $(\forall x, y, a)(T(x, y, a) \to M(x, a))$

[12 marks]

Answer to (a)

(i) If one person talks to another about something then the other talks to the one about it too.
(ii) $(\forall x, a)\neg T(x, x, a)$
(iii) $(\forall a, b)(F(a) \wedge F(b) \ \to a = b)$
(iv) $(\forall a)(\exists x, y)(M(x, a) \wedge M(y, a) \wedge x \neq y)$
(v) Any society with at least two members has two (distinct) members one of which talks to the other about a society involving fighting.
(vi) Anyone who talks to anyone about a society is a member of that society.

(b) Is it possible to satisfy (i)–(vi) simultaneously? Either give a concrete definition of two sets I and S and relations M, F and T for which (i)–(vi) are all true or prove that you can derive a contradiction from (i)–(vi).

[4 marks]

Answer to (b)

It is consistent. There is precisely one society, and it involves fighting. It has two members who talk to each other (but not themselves) about fighting.

There is actually another solution, which i have only just noticed, and which i am pretty sure the examiners did not intend... There are no societies and no people! This is possible because none of the axioms say that there are any societies or people at all!

(c) Here are several attempts to formalise "Somebody talks about everything". Explain what they actually mean, discussing whether or not each is a reasonable formalisation.

(i) $(\exists x)(\forall a)(\exists y)T(x, y, a)$
(ii) $(\exists x)(\exists y)(\forall a)T(x, y, a)$
(iii) $(\forall x)(\forall a)(\exists y)T(x, y, a)$
(iv) $(\exists y)(\forall a)(\forall x)T(x, y, a)$

[4 marks]

Answer to (c)

(i) There is someone who, for every topic, has someone to talk to about it
(ii) Someone talks to someone about everything
(iii) For every topic and every person, there is someone that person talks to about that topic
(iv) There is someone whom everybody talks to about everything