2009p1q4

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I won't insult my readers' intelligence by offering answers to part (a); it's bookwork. Part (b) is also beneath their notice. The person who set parts (c) and (d) is probably a former supervisee of mine, since this is [the easy part of] an old example sheet question of mine. In contrast part (e) does need a *wee* bit of attention.

For part (i) reflect that a union of irreflexive relations is obviously irreflexive. Reflect also that being a symmetric relation (if you are a set of ordered pairs) is being closed under a particular unary operation ("flip!"). If R and S are both closed under a unary operation then so is $R \cup S$.

No similar argument is going to work for part (ii), and with a bit of ingenuity the reader should be able to come up with a counterexample.

(e) part (iii) repays thought. The relation Q being defined on the power set of A *must* be symmetric because if you swap 'X' and 'Y' in the formula that defines it then you obtain an alphabetic variant of the original formula. In slang one says that the formula is *symmetric in 'X' and 'Y'*.

In the medium term (but **not** in the short term!) it might be an idea to think about why this is a proof (or at least a recipe for a proof), and how one might turn it into a proof. Perhaps one should think of it as a (meta) proof that there is a proof.

Finally Q is not irreflexive, beco's the empty set is related to itself. Really? Yes: everything in the empty set is related to everything in the empty set—vacuously! If you have worries about $\forall x F(x)$ being true in the empty domain you've just got to stop worrying. This is beco's, if you *were* planning to worry about it, you would have to explain what it is about the various Fs that makes $\forall x F(x)$ true for those Fs that make it true that is different from those Fs for which it comes out false. (Or do you want it to be always false...? Don't go there.)