153

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What is 2^{153} mod 153?

The obvious thing to try is to see if some small powers of 2 are close to a multiple of 153. Well, maybe there some are but i for one couldn't find any. So let's see if there is anything else we can do. When in doubt factorise everything in sight...

$153 = 17 \cdot 9$ so let's calculate 2^{153} mod these two bases and see what happens.

$2^8 = 256 = 1 + 255 = 3 \cdot 5 \cdot 17$ whence 2^8 is congruent to 1 mod 17. 152 is a multiple of 8 so 2^{152} is congruent to 1 mod 17 and 2^{153} is congruent to 2 mod 17.

Also 2^6 is congruent to 1 mod 9, 150 is a multiple of 6 so 2^{150} is congruent to 1 mod 9, and 2^{153} is congruent to 8 mod 9.

So what is 2^{153} mod 153? Well, it's a number less than 153 so it's a certain number (say x) of blocks of size 9 (because 9|153) with 8 left over (beco's 2^{153} is congruennt to 8 mod 9). So it must be $9x + 8$ for some x Similarly it must be $17y + 2$ for some y (because 17|153). We get 53 if $y = 3$ and $x = 4$ so it *could* be 53. To be sure that it *is* 53 we just have to check that $y = 3; x = 5$ is the only solution to $9x + 8 = 17y + 2$ that is less than 153. I'm leaving that to you for the moment!

[Thank you Marta Kuczma mmk64 for spotting a typo!]